Nonlinear PML for Absorption of Nonlinear Electromagnetic Waves

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Abstract—Nonlinear time-domain numerical modeling requires the development of absorbing boundary conditions to effectively absorb the nonlinear electromagnetic waves. In this paper, based on Berenger's PML, the nonlinear perfectly matched layer (nPML) absorbing condition is developed and implemented in the recently proposed TLM-based FDTD method. Numerical results show the effectiveness of the nPML. The proposed nPML scheme can also be implemented to others FD-TD schemes.

I. INTRODUCTION

Because nonlinearity has potential applications such as all-optical signal processing (see e.g. [1]-[3] and references therein), it has received growing interest [1]-[10][13][14] in applying it to the design of novel nonlinear structures for high-speed communications. However, the analytical solutions for such nonlinear structures are in general difficult to find. Therefore, numerical techniques, in both frequency-domain [3]-[7] and time-domain [1][8]-[10][13][14] have to be employed. Among the numerical techniques in time-domain are various versions of FDTD schemes (see [1][8][13][14] and references therein).

In order to apply these FDTD methods to open nonlinear structures, an appropriate absorbing boundary condition (ABC), which can effectively absorb nonlinear electromagnetic waves, needs to be developed. As a result, an infinite computation domain can be truncated with the nonlinear ABCs, allowing a practical numerical simulation of an open nonlinear structure.

Since the initial work by Berenger on the perfectly matched layer (PML) [11], various numerical experiments have been performed. The PML has been demonstrated to be the most effective ABC for linear electromagnetic wave propagation so far [1][12] (and references therein).

A standard PML consists of lossy layers with both electric and magnetic conductivities [11]. By appropriately selecting constitutive parameters, an extremely-low-reflection from PML layers is achieved while the waves are attenuated inside the PML region. Variations, or improvements of the PML, have been reported, e.g. [15]. However, all of the PML schemes developed so far are limited to the linear wave absorption. They can not be applied directly to nonlinear open structures without modifications.

In this presentation, the standard PML is adopted to absorb nonlinear electromagnetic waves by applying nonlinearity directly to the PML

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permittivity and permeability. The resultant nonlinear PML (nPML) scheme is then implemented in the recently proposed TLM based FDTD scheme [13]. The numerical results show the effectiveness of nPML. The nPML can also be easily implemented in other FDTD schemes such as Yee's scheme [1].

II. Nonlinear PML (nPML)

For simplicity, consider an one-dimensional Kerr-like nonlinear medium with the following nonlinear refractive index [1][3][5][13][14]:

$$n = n_0 + n_2|\mathbf{E}|^2 \qquad (1)$$

where n_0 is the linear part of the refractive index and n_2 is the nonlinear coefficient of the medium. Generally, the nonlinear term of (1) is very small compared to the linear part [1][3][14]. Therefore, relative dielectric constant of the nonlinear medium can be obtained approximately as follows [1][3][14]:

$$\varepsilon_r = n^2 \approx n_0^2 + 2n_0n_2|\mathbf{E}|^2 \qquad (2)$$

To truncate the computing domain of the nonlinear medium, a nonlinear PML is constructed. Like the standard linear PML, the match conditions, which dictate the relationships among the electric conductivities, magnetic conductivities, permittivities, and permeabilities, are still enforced in the nPML regions. The difference is that the medium is now nonlinear. Therefore, nonlinearity (1) should also be imposed on the PML permittivity, which will then change with the electric field intensity. In the other words, the following modified match conditions have to be satisfied simultaneously in the nPML region:

$$\frac{\sigma_z}{\varepsilon_0\varepsilon_r} = \frac{\sigma_z^*}{\mu} \qquad (3)$$

$$\varepsilon_r = n^2 \approx n_0^2 + 2n_0n_2|\mathbf{E}|^2 \qquad (4)$$

Note that the permittivity of a nPML medium is now field-intensity dependent (and therefore location dependent). Equation (4) can be solved with a FDTD recursive algorithm.

To further reduce the reflections, instead of terminating a perfect conducting wall in the last layer of the PML as done in a standard PML, a resistance wall with E/H equal to the wave impedance of the last cell [13] is used to terminate the nPML.

III. Numerical Results

To effect comparison, an one-dimensional nonlinear medium terminated at both ends with nPML is simulated using the TLM based FDTD method [13]. A spatial pulse excitation at $t=0$ is used. To ensure that a nonlinear wave is indeed excited, reference simulations without PMLs for linear and nonlinear media of the same dimensions, the same initial excitation, and the same grids were run, respectively. Figure 1(a) shows the simulation results of the linear and nonlinear media terminated only with the same resistance wall (no PML terminations).

As can be seen, a nonlinear wave is indeed excited since the linear and nonlinear waves are shown to behaviour differently. The significant part of the differences exist especially for the reflected waves. For the linear waves, both the incident and reflected waves are symmetric because the initial spatial excitation is symmetric. However, for the nonlinear wave, this symmetry breaks down. The forward- and reflected waves are *asymmetric*. In addition, the magnitude of the reflected nonlinear wave is larger than that of the linear wave.

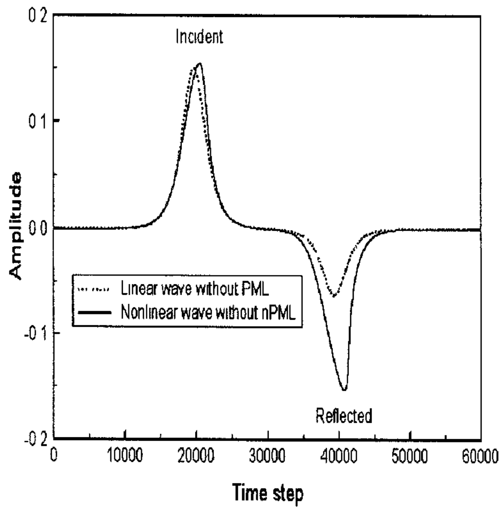

Figure 1(a) Comparisons of linear and Nonlinear waves.

After the existence of the nonlinear wave was numerically confirmed, PML and nPML were added in between the computation domain and the resistance wall for linear and nonlinear waves, respectively. Figure 1(b) shows the simulated results.

As can be seen, the reflected waves for both the linear and nonlinear cases become invisible. This indicates that the linear wave is absorbed by the linear PML and the nonlinear wave is absorbed by the proposed nPML. The effectiveness of the nPML is thus shown.

For a quantitative demonstration of the effectiveness of the nPML, Figure 2 shows the magnitude of the reflected waves from nPML versus time, recorded at a fixed spatial point in the computation domain. It can be seen that the amplitude of the reflected nonlinear wave is very small in comparison to the incident wave. The nPML did absorb the nonlinear wave very effectively.

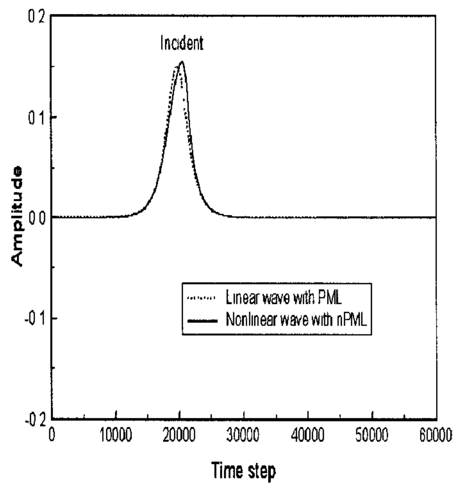

Figure 1(b) Simulation results with linear and nonlinear PMLs.

IV. Conclusion

A nonlinear perfectly matched layer (nPML) scheme is presented in this paper for the effective absorption of nonlinear electromagnetic waves propagating in a Kerr-like nonlinear medium. Numerical experiments demonstrate its validity and effectiveness. Although the newly proposed nPML was implemented in the TLM based FDTD grid for one-dimensional cases, it can be easily extended to multi-dimensional problems and implemented in other FDTD schemes such as standard Yee's scheme. The investigation along this line is currently under way in our laboratory.

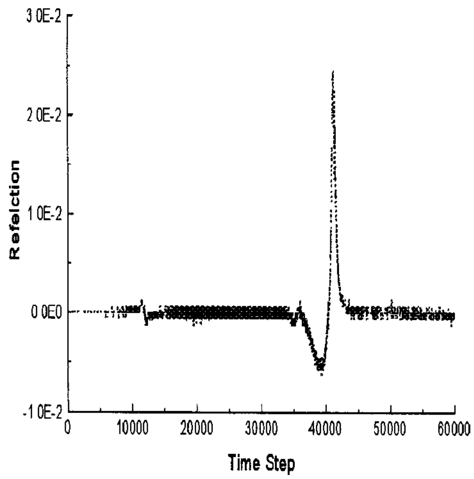

Figure 2: The reflected nonlinear wave vs time (recorded at a fixed spatial point). The vertical axis represents the ratio of the reflected wave normalized to the maximum magnitude of the incident wave.

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